Subject Information Guide

Geometry and symmetry of nonlinear integrable equations

Semester 2, 2019

Administration and contact details

Host Department	College of science and engineering
Host Institution	Flinders University
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Subject details

Handbook entry URL	Click here to enter text.
Subject homepage URL	https://www.flinders.edu.au/webapps/stusys/index.cfm/topic/main?subj=MATH&numb=7720&year=2019
Honours student hand-out URL	Click here to enter text.
Start date:	29 July 2019
End date:	30 October 2019
Contact hours per week:	2.
Lecture day and time:	Monday, Wednesday, 11-12
Description of electronic access arrangeme	**Access grid room**

nts for students (for example, WebCT)	

Subject content

1. Subject content description

The study of integrable equations originated from that of nonlinear partial differential equations which admit soliton solutions, sometimes referred to as "soliton equations". Now days, intergrable equations include both ordinary and partial, differential and difference equations.
One of the many remarkable properties that characterise what we call integrable equations is that there exist transformations between different solutions of the equation, called Backlund transformations. However, to find such transformations amounted to something of "a black art" and it was not completely understood why they should even exist.

It is by now well established that the deep reason for the existence of such transformations is given by the underlying symmetries of the equations, which can be completely characterized using the description of Weyl groups (crystallographic reflection groups). The Weyl groups give the discrete symmetries of the differential (or continuous) equations. Moreover they give rise to discrete equations which are now understood to be more fundamental than their continuous counterparts.

This course introduces geometric and symmetric characterizations of a variety of discrete integrable equations. After first motivated by their origins from continuous integrable equations such as the soliton equations and the classical Painlevé equations, we launch into the theories of difference equations, developing the basic tools of root systems and corresponding finite and affine Weyl groups. We introduce some remarkable geometric and combinatorial properties of these groups relevant in our context of discrete integrable equations. We shall see that the Weyl group description of nonlinear discrete equations not only enables us to study the properties of such equations, but more importantly it allows us to uncover connections between different classes of discrete integrable equations hence providing us with a clearer picture of this area of study, which is an important branch of mathematical physics.

2. Week-by-week topic overview

- week 1: Integrable PDEs and the Painleve equations
- week 2: Partial difference equations (PΔEs) arising from PDEs (KdV, KP)

- week 3: Multi-dimensional consistency and classifications of PΔEs (l-KdV, dKP)
- week 4: Discere Painlevé equations arising from the Painlevé equations
- week 5: Revision of linear algebra and introduce the Root systems
- week 6: Geometric realizations of the Weyl groups
- week 7: Symmetry and geometric interpretations of the l-KdV equation
- week 8: Symmetry and geometric interpretations of the dKP equation
- week 9: Affine Weyl groups and the weight lattice
- week 10: Painlevé equations with affine Weyl group symmetries
- week 11:Discere Painlevé equations
- week 12: Reductions of a l-KdV equation to a discrete Painlevé equation

3. Assumed prerequisite knowledge and capabilities

Basic differential equations and linear algebra.

4. Learning outcomes and objectives

Outcome 1: Understand transformations that leaves nonlinear differential equations invariant and their characterizations as symmetry groups
Outcome 2: Be familiar with the root system and the combinatorial and geometric properties of the finite and affine Weyl groups
Outcome 3: Understand the Weyl group description of integrable equations
Outcome 4: Able to interpret properties of the Weyl group in the context of integrable equations

AQF specific Program Learning Outcomes and Learning Outcome Descriptors (if available):

AQF Program Learning Outcomes addressed in this subject	Associated AQF Learning Outcome Descriptors for this subject
Outcome 1	K1, S1, S2, S5, A2, A3, A4
Outcome 2	K1, S1, S2, S5, A2, A3, A4
Outcome 3	K1, S1, S2, S5, A2, A3, A4
Outcome 4	K1, S1, S2, S5, A2, A3, A4

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Learning Outcome Descriptors at AQF Level 8

Knowledge

K1: coherent and advanced knowledge of the underlying principles and concepts in one or more disciplines

K2: knowledge of research principles and methods

Skills

S1: cognitive skills to review, analyse, consolidate and synthesise knowledge to identify and provide solutions to complex problem with intellectual independence

S2: cognitive and technical skills to demonstrate a broad understanding of a body of knowledge and theoretical concepts with advanced understanding in some areas

S3: cognitive skills to exercise critical thinking and judgement in developing new understanding

S4: technical skills to design and use in a research project

S5: communication skills to present clear and coherent exposition of knowledge and ideas to a variety of audiences

Application of Knowledge and Skills

A1: with initiative and judgement in professional practice and/or scholarship

A2: to adapt knowledge and skills in diverse contexts

A3: with responsibility and accountability for own learning and practice and in collaboration with others within broad parameters

A4: to plan and execute project work and/or a piece of research and scholarship with some independence

5. Learning resources

Hietarinta, J., Joshi, N., & Nijhoff, F., *Discrete Systems and Integrability*, Cambridge University Press, 2016.

Noumi, M., *Painlevé equations through symmetry*, American Mathematical Society, Providence, R.I., USA, 2004.

Humphreys, J., *Reflection Groups and Coxeter Groups*, Cambridge: Cambridge University Press, 1990.

6. Assessment

Exam/assignment/classwork breakdown					
Exam	40 %	**Assignment**	60 %	**Class work**	
Assignment due dates					
Assignments will be given in weeks: 3, 6, 9					
Approximate exam date				30th October	

Institution Honours program details

Weight of subject in total honours assessment at host department	**10% (students choose 6 subjects, each worth 10%)**
Thesis/subject split at host department	**thesis is worth 40%, and subjects 60%**
Honours grade ranges at host department:	
H1	80-100 %
H2a	75-79 %
H2b	70-74 %
H3	65-69 %