AMSI
AUSTRALIAN MATHEMATICAL SCIENCES INSTITUTE

ACE Network Subject Information Guide

Optimal Transportation and Monge-Ampere Equations

Semester 2, 2019

Administration and contact details

Host Department	School of Mathematics and Applied Statistics
Host Institution	University of Wollongong
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Subject details

Handbook entry URL	TBA
Subject homepage URL	TBA
Honours student hand-out URL	TBA
Start date:	TBA
End date:	TBA
Contact hours per week:	TBA
Lecture day and time:	TBA
Description of electronic access arrangements for students (for example, WebCT)	Resources will be hosted and available for download from the Lecturer's website. Details will be given at the commencement of the course.

Subject content

1. Subject content description

This subject gives an introduction to the optimal transportation problem, which arises in a broad range of areas: Fluid Mechanics; Partial Differential Equations (PDE); Optimisation; and Financial Mathematics. From an analytic point of view, this subject introduces the elementary existence and uniqueness theory, with a focus on recent development on regularity theory. It involves the study of the Monge-Ampere type PDE, whose applications extend to more areas, in particular, in calculus of variations and geometry.

2. Week-by-week topic overview

The first half (6 weeks) is about the existence and uniqueness results:
Week 1: Introduction to optimal transportation
Week 2: Kantorovich's duality and linear optimisation
Week 3: Existence of optimal mappings
Week 4: Geometric characterisation of optimal maps
Week 5: Basics of convex analysis
Week 6: Brenier's factorisation theorem

The second half (6 weeks) is about the regularity theory:
Week 7: Introduction to Monge-Ampere equations
Week 8: Comparison principle and boundary value problem
Week 9: A priori estimates and boundary estimates
Week 10: Pogorelov type estimates
Week 11: Regularity of optimal mappings I
Week 12: Regularity of optimal mappings II

3. Assumed prerequisite knowledge and capabilities

Multivariable calculus, Linear algebra, and basic PDEs

4. Learning outcomes and objectives

After successful completion of this subject, students should be able to perform the following tasks:

AQF specific Program Learning Outcomes and Learning Outcome Descriptors (if available):

AQF Program Learning Outcomes addressed in this subject	Associated AQF Learning Outcome Descriptors for this subject
1. Define, understand and utilise some key concepts about the optimal transportation problem;	K1, S1, A1
2. Formulate and solve problems of Kantorovich's dual linear optimisation;	K1, K2, S1, S2, S4, A1, A2
3. Understand and appreciate some fundamental theorems and proofs in fully nonlinear PDEs;	K1, K2, S1, S5, A3
4. Compute and analyse basic types of a priori estimates;	K2, S1, S2, S3, S4, S5, A2
5. Apply ideas from a priori estimates in the contexts of Monge-Ampere type equations;	K2, S2, S3, S4, A1, A2, A4

6. Establish the regularity of optimal mappings with the quadratic cost function;	K1, S1, S3, S4, S5, A3, A4
7. Clearly present mathematical concepts relevant to the subject in written form, demonstrating skill in constructing clear mathematical arguments.	K2, S3, S4, S5, A1, A2, A4

Learning Outcome Descriptors at AQF Level 8
Knowledge
K1: coherent and advanced knowledge of the underlying principles and concepts in one or more disciplines
K2: knowledge of research principles and methods
Skills
S1: cognitive skills to review, analyse, consolidate and synthesise knowledge to identify and provide solutions to complex problem with intellectual independence
S2: cognitive and technical skills to demonstrate a broad understanding of a body of knowledge and theoretical concepts with advanced understanding in some areas
S3: cognitive skills to exercise critical thinking and judgement in developing new understanding
S4: technical skills to design and use in a research project
S5: communication skills to present clear and coherent exposition of knowledge and ideas to a variety of audiences
Application of Knowledge and Skills
A1: with initiative and judgement in professional practice and/or scholarship
A2: to adapt knowledge and skills in diverse contexts
A3: with responsibility and accountability for own learning and practice and in collaboration with others within broad parameters
A4: to plan and execute project work and/or a piece of research and scholarship with some independence

5. Learning resources

Reference books:

Gilbarg, D. and Trudinger, N. S.: *Elliptic partial differential equations of second order*. Reprint of the 1998 edition, Classics in Mathematics, *Springer-Verlag, Berlin*, 2001.

Villani, C.: *Topics in optimal transportation*, vol. 58 of Graduate Studies in Mathematics. *Amer. Math. Soc., Providence, RI*, 2003.

Villani, C.: *Optimal transport. Old and New*, Grundlehren Math. Wiss. 338, *Springer-Verlag, Berlin*, 2009.

Reference notes:

Ambrosio, L.: Lecture notes on optimal transport problems. *Mathematical aspects of evolving interfaces (Funchal, 2000),* 1—52, Lecture Notes in Math., 1812*, Springer, Berlin*, 2003.

Caffarelli, L. A.: The Monge-Ampere equation and optimal transportation, an elementary review. *Optimal transportation and applications,* 1—10, Lecture Notes in Math., 1813, *Springer, Berlin*, 2003.

Evans, L. C.: Partial differential equations and Monge-Kantorovich mass transfer. In *Current developments in mathematics*, 1997 (Cambridge, MA), 65—126, Int. Press, Boston, MA, 1999.

Urbas, J.: Mass transfer problem, *Lecture Notes,* Univ. of Bonn, 1998.

Some of above references are available in the library or on internet.

6. Assessment

Exam/assignment/classwork breakdown					
Exam	50 %	**Assignment**	50 %	**Class work**	
Assignment due dates		TBA	TBA		
Approximate exam date				TBA	

Institution Honours program details

Weight of subject in total honours assessment at host department	1/8
Thesis/subject split at host department	BMath (Hons): Thesis worth 25% BMathAdv (Hons): Thesis worth 37.5%
Honours grade ranges at host department:	
H1	85-100
H2a	75-84
H2b	65-74
H3	50-64