ACE Network Subject Information Guide

MATH4101 Introduction to Valued Fields

Semester 1, 2019

Administration and contact details

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Subject details

Handbook entry URL	https://www.newcastle.edu.au/course/MATH4101
Subject homepage URL	TBC
Honours student hand-out URL	The initial course notes can be found here: http://math.uga.edu/~pete/8410FULL.pdf Additional notes will be posted as we go.
Start date:	26 Feb 2019
End date:	4 June 2019
Contact hours per week:	2
Lecture day and time:	Tuesdays 11am – 1pm (Newcastle time)
Description of electronic access arrangements for students (for example, WebCT)	A course website will be created soon where students can download lecture notes and assignments

Subject content

1. Subject content description

On the field of rational numbers, besides the usual notion of absolute value, one can also define a p-adic absolute value for each prime p which measures divisibility by p. Essentially, a rational number in lowest terms has small p-adic absolute value if its numerator is highly divisible by p. Just like the usual absolute value, the p-adic absolute value is multiplicative, but satisfies a stronger form of the triangle inequality called the ultrametric inequality.
The usual absolute value defines a metric on the field of rational numbers, whose completion is the field of real numbers. Similarly, completion with respect to a p-adic absolute value gives the field of p-adic numbers. The analytic theory of p-adic numbers is rich and interesting, and in many cases one obtains stronger results than in real or complex analysis, due to the stronger ultrametric inequality.
This course will study the general theory of absolute values on fields and the resulting topologies and analysis. These foundations lead to important concepts in number theory (such as the behaviour of primes in field extensions) and the theory of topological groups, such as totally disconnected locally compact (t.d.l.c.) groups.
Besides the intrinsic interest of this basic area of mathematics, this course serves as an introduction to research topics of current interest at the University of Newcastle and elsewhere.

2. Week-by-week topic overview

This course is based on lecture notes by Pete L. Clark, UGA. We will cover the first part of Clark's notes:

- Basic theory of absolute values, including Artin-Whaples Approximation
- Completion of normed fields
- Extension of absolute values
- Structure theory of locally compact fields: unramified, tamely ramified and wildly ramified extensions

After that, depending on how much time is left, we will cover basic topics on topological groups and t.d.l.c. groups.

3. Assumed prerequisite knowledge and capabilities

Basic familiarity of abstract algebra will be assumed: Groups, rings, fields, field extensions. Further concepts from commutative algebra will make an appearance, such as local rings, Dedekind domains and tensor products, but the basic properties of these objects will be recalled as necessary.

4. Learning outcomes and objectives

- Demonstrate an understanding of the content and context of valued fields
- Apply advanced mathematical problem solving skills
- Use sophisticated mathematical communication skills in the presentation of mathematical arguments

AQF specific Program Learning Outcomes and Learning Outcome Descriptors (if available):

AQF Program Learning Outcomes addressed in this subject	Associated AQF Learning Outcome Descriptors for this subject
Insert Program Learning Outcome here	Choose from list below
Insert Program Learning Outcome here	Choose from list below
Insert Program Learning Outcome here	Choose from list below
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Insert Program Learning Outcome here	Choose from list below
Insert Program Learning Outcome here	Choose from list below
Insert Program Learning Outcome here	Choose from list below

Learning Outcome Descriptors at AQF Level 8

Knowledge

K1: coherent and advanced knowledge of the underlying principles and concepts in one or more disciplines

K2: knowledge of research principles and methods

Skills

S1: cognitive skills to review, analyse, consolidate and synthesise knowledge to identify and provide solutions to complex problem with intellectual independence

S2: cognitive and technical skills to demonstrate a broad understanding of a body of knowledge and theoretical concepts with advanced understanding in some areas

S3: cognitive skills to exercise critical thinking and judgement in developing new understanding

S4: technical skills to design and use in a research project

S5: communication skills to present clear and coherent exposition of knowledge and ideas to a variety of audiences

Application of Knowledge and Skills

A1: with initiative and judgement in professional practice and/or scholarship

A2: to adapt knowledge and skills in diverse contexts

A3: with responsibility and accountability for own learning and practice and in collaboration with others within broad parameters

A4: to plan and execute project work and/or a piece of research and scholarship with some independence

5. Learning resources

The initial course notes can be found here:
http://math.uga.edu/~pete/8410FULL.pdf

Additional notes will be posted as we go.

Useful books include:

- F. Q. Gouvea: “p-adic numbers”, Universitext, Springer-Verlag, 1993. This is a very elementary introduction to some of the basic concepts studied in this course.
- J.-P. Serre: “Local Fields”, Graduate Texts in Mathematics 67, Springer-Verlag, 1979. This is a more advanced textbook. Our

Our course will be at a level roughly between these two books, with some extra material on topological groups added at the end.

6. Assessment

Exam/assignment/classwork breakdown					
Exam	50 %	**Assignment**	50 %	**Class work**	0 %
Assignment due dates		9 April 2019	28 April		
Approximate exam date				Mid June 2019	

Institution Honours program details

Weight of subject in total honours assessment at host department	10 of 80 units
Thesis/subject split at host department	30 of 80 units pertain to an honours project
Honours grade ranges at host department:	
HD	85-100
D	75-84
C	65-74
P	50-64