Subject Information Guide

Asymptotic Methods for Applied Differential Equations – MATH707

Semester 1, 2018

Administration and contact details

Host Department	Department of Mathematics
Host Institution	Macquarie University
Name of lecturer	Justin Tzou
Phone number	02 9850 8925
Email Address	justin.tzou@mq.edu.au
Homepage	http://web.science.mq.edu.au/~jtzou
Name of Honours coordinator	Dr Adam Sikora
Phone number	
Email Address	adam.sikora@mq.edu.au

Subject details

Handbook entry URL	http://www.handbook.mq.edu.au/2018/Units/ResearchUnit/MATH707
Subject homepage URL	http://web.science.mq.edu.au/~jtzou/TEACHING/MATH707.html
Honours student hand-out URL	http://web.science.mq.edu.au/~jtzou/TEACHING/MATH707.html
Start date:	26 February 2018
End date:	1 June 2018
Contact hours per week:	2 hours per week
Lecture day and time:	TBA
Description of electronic access arrangements for students (for example, WebCT)	Materials will be made available on lecturer's website.

Subject content

1. Subject content description

Various physical phenomena in nature can be modelled by ordinary and partial differential equations. These equations rarely have exact solutions, while numerical solutions may be uninformative and time-consuming to compute. Asymptotic and perturbation methods can

often be employed to obtain approximate solutions in physically significant parameter regimes. The aim of this course is to present students with the necessary tools to understand and analyse recent advances in asymptotic methods for differential equations pertaining to applications such as animal markings, crime hot spots, and intracellular transport.

2. Week-by-week topic overview

The following content will be distributed over the 12-week semester:

- **Introduction**
  - introduction to asymptotic series
  - perturbed algebraic equations

- **Regular perturbations**
  - eigenvalues of perturbed boundary value problems; solvability conditions
  - boundary value problems on perturbed domains
  - nonlinear initial value problems
  - stability analysis of solutions of parabolic PDEs

- **Singular perturbations**
  - matched asymptotic methods for boundary value problems
    - boundary layers
    - internal layers
    - nested layers
  - WKB method; turning points

- **Recent research and applications: asymptotic methods for PDEs**
  - weakly nonlinear analysis of parabolic PDEs with applications in
    - crime hotspot suppression
    - vortex dynamics in Bose-Einstein condensates
  - matched asymptotic methods for singularly perturbed parabolic PDEs in one spatial dimension
    - analysis of slow dynamics and stability of fully nonlinear localised patterns of reaction-diffusion systems
  - hybrid asymptotic-numerical techniques for singularly perturbed parabolic and elliptic PDEs in two and three spatial dimensions
    - analysis of slow dynamics and stability of fully nonlinear localised spot patterns of reaction-diffusion systems
    - optimising the principal eigenvalue of the Laplacian on a punctured two-dimensional domain
    - narrow escape and narrow capture problems

3. Assumed prerequisite knowledge and capabilities

Differential equations (as obtained through, for example, a good third year unit on differential equations; both ODEs and PDEs). Basic knowledge of the following may be beneficial:

- MATLAB (programming language)
- linear algebra
- numerical methods for differential equations

4. Learning outcomes and objectives

At the end of this course, students will be able to understand and apply asymptotic methods for analysing various classes of differential equations. Moreover, they will gain knowledge of the current state of research with regards to applications of asymptotic methods in certain areas of applied mathematics.

AQF specific Program Learning Outcomes and Learning Outcome Descriptors (if available):

AQF Program Learning Outcomes addressed in this subject	Associated AQF Learning Outcome Descriptors for this subject
K1	coherent and advanced knowledge of the underlying principles and concepts in one or more disciplines
K2	knowledge of research principles and methods
S1	cognitive skills to review, analyse, consolidate and synthesise knowledge to identify and provide solutions to complex problem with intellectual independence
S2	cognitive and technical skills to demonstrate a broad understanding of a body of knowledge and theoretical concepts with advanced understanding in some areas
S3	cognitive skills to exercise critical thinking and judgement in developing new understanding
S4	technical skills to design and use in a research project
A2	to adapt knowledge and skills in diverse contexts

5. Learning resources

Written lecture notes will be available for students and will be posted on the unit website. While not necessary, the following references and resources may be useful:

- *Advanced Mathematical Methods for Scientists and Engineers*, C.M. Bender and S.A. Orszag, McGraw-Hill (1978).
- *Asymptotic Analysis*, J.D. Murray, Spring-Verlag (1984).

- *Multiple Scale and Singular Perturbation Methods*, J. Kevorkian and J.D. Cole, Springer-Verlag (1996).
- *Introduction to Perturbation Methods*, M.H. Holmes, Springer-Verlag (1998).
- MATLAB (any version) or a free alternative such as GNU Octave

6. Assessment

Exam/assignment/classwork breakdown					
Exam	60%	**Assignment**	40%	**Class work**	0%
Assignment due dates		March 26, 2018 (TBC)	April 30, 2018 (TBC)	May 21, 2018 (TBC)	N/A
Approximate exam date				TBC	

Institution Honours program details

Weight of subject in total honours assessment at host department	12.5% of BPhil
Thesis/subject split at host department	BPhil has no thesis; thesis is 90% of MRes
Honours grade ranges at host department:	
H1	85%
H2a	75%
H2b	65%
H3	50%