AMSI
AUSTRALIAN MATHEMATICAL SCIENCES INSTITUTE

Subject Information Guide

Integrable Systems AMH2

Semester 2, 2018

Administration and contact details

Host Department	College of Science and Engineering
Host Institution	Flinders University
Host Department	School of Mathematics and Statistics
Host Institution	University of Sydney
Name of lecturer	Nalini Joshi
Phone number	02 9351 2172
Email Address	nalini.joshi@sydney.edu.au
Homepage	http://sydney.edu.au/science/people/nalini.joshi.php
Name of lecturer	Milena Radnovic
Phone number	02 9351 5782
Email Address	milena.radnovic@sydney.edu.au
Homepage	http://sydney.edu.au/science/people/milena.radnovic.php
Name of lecturer	Yang Shi
Phone number	08 8201 3749
Email Address	yang.shi@flinders.edu.au
Homepage	http://www.flinders.edu.au/people/yang.shi
Name of Honours coordinator	Robert Marangell
Phone number	02 9351 5795
Email Address	robert.marangell@sydney.edu.au

Subject details

Handbook entry URL	http://www.maths.usyd.edu.au/u/UG/HM/coordinator/applied2018.pdf
Subject homepage URL	http://www.maths.usyd.edu.au/u/UG/HM/AMH2/
Honours student hand-out URL	
Start date:	30 July 2018

End date:	2 November 2018
Contact hours per week:	**2**
Lecture day and time:	
Description of electronic access arrangements for students (for example, WebCT)	**Access grid room.**

Subject content

1. Subject content description

The mathematical theory of integrable systems has been described as one of the most profound advances of twentieth century mathematics. They lie at the boundary of mathematics and physics and were discovered through a famous paradox that arises in a model devised to describe the thermal properties of metals (called the Fermi-Pasta-Ulam paradox).

In attempting to resolve this paradox, Kruskal and Zabusky discovered exceptional properties in the solutions of a non-linear PDE, called the Korteweg-de Vries equation (KdV). These properties showed that although the solutions are waves, they interact with each other as though they were particles, i.e., without losing their shape or speed, until then thought to be impossible for solutions of non-linear PDEs. Kruskal invented the name *solitons* for these solutions.

Solitons are known to arise in other non-linear PDEs and also in partial difference equations. These systems and their symmetry reductions are now called *integrable systems*. These systems occur as universal limiting models in many physical situations.

This course introduces the mathematical properties of such systems. In particular, we will study their solutions, symmetry reductions called the Painlevé equations and their discrete versions. It focuses on mathematical methods created to describe the solutions of such equations and their interrelationships.

The course also includes discrete integrable systems. While having many properties common with those of the continuous (differential) systems, it is now understood that they are more fundamental.

First we make contact with the continuous systems, using as a motivating example, the modified KdV (mKdV) equation deriving its discrete analogue, namely the lattice mKdV (l-mKdV) equation. We then go on studying several classifications of nonlinear discrete integrable systems, of their integrable properties, such as Lax pairs, special solutions, their geometric and combinatorial interpretations.

A key theme of this part of the course is the discrete symmetry of these systems. They form what are known as reflection groups. We develop the basic theories of root systems and their corresponding reflection groups, and their application in the context of discrete integrable systems.

We will see that the group description of nonlinear discrete equations not only enables us to study the properties of such equations, more importantly it clarifies the connections between different classes of discrete integrable systems.

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2. Week-by-week topic overview

- week 1: KdV equation
- week 2: Schrodinger operator
- week 3: scattering theory for Schrodinger operator
- week 4: inverse scattering
- week 5: reflectionless potential
- week 6: Darboux transformations
- week 7: Partial difference equations (PΔEs) arising from PDEs
- week 8: Consistency around the cube and classifications of PΔEs
- week 9: Root systems and Reflection groups
- week 10: Painlevé equations as symmetry reductions PDEs
- week 11:Discere Painlevé equations
- week 12: Use of reflection groups in the context of discrete integrable systems

3. Assumed prerequisite knowledge and capabilities

Basic differential equations.

4. Learning outcomes and objectives

Outcome 1: Understand the inverse scattering transform method: how to use it to solve integrable systems and find solitons; how to prove that it works for certain initial conditions.

Outcome 2: Understand the transformation theory that relates integrable systems to each other and the reductions from PDEs to ODEs.

Outcome 3: Understand how to use symmetry groups to describe transformations, find special solutions, recurrence relations and related discrete integrable systems.

Outcome 4: Describe other properties of solutions of integrable systems, in particular behaviours that occur in limits.

AQF specific Program Learning Outcomes and Learning Outcome Descriptors (if available):

AQF Program Learning Outcomes addressed in this subject	**Associated AQF Learning Outcome Descriptors for this subject**
Outcome 1	K1, S1, S2, S5, A2, A3, A4
Outcome 2	K1, S2, A2
Outcome 3	K1, S1, S2, S5, A2, A3, A4
Outcome 4	K1, S1, S2, S5, A2, A3, A4

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5. Learning resources

Useful texts include:

M. J. Ablowitz and H. Segur, Solitons and the inverse scattering transform, SIAM, Philadelphia, USA, 1981.

M. J. Ablowitz and P.A. Clarkson, Solitons, nonlinear evolution equations and inverse scattering, Cambridge University Press, Cambridge, UK, 1991.

P.G. Drazin and R.S. Johnson, Solitons : an introduction, Cambridge University Press, Cambridge, UK, 1989.

M. Noumi, Painlevé equations through symmetry, American Mathematical Society, Providence, R.I., USA, 2004.

There are many interesting links on solitons, including the Wikipedia pages on Solitons and the Korteweg-de Vries equation.

Lecture notes and useful links will be provided on the web-page.

6. Assessment

Exam/assignment/classwork breakdown					
Exam	40 %	**Assignment**	60 %	**Class work**	
Assignment due dates		The assignments will be given in weeks 2-11, each due the following week			
Approximate exam date				mid November 2018	

Institution Honours program details

Weight of subject in total honours assessment at host department	**10% (students choose 6 subjects, each worth 10%)**
Thesis/subject split at host department	**thesis is worth 40%, and subjects 60%**
Honours grade ranges at host department:	
H1	80-100 %
H2a	75-79 %
H2b	70-74 %
H3	65-69 %