AMSI
AUSTRALIAN MATHEMATICAL
SCIENCES INSTITUTE

Subject Information Guide

Algebra (Algebra MATH702)

Semester 1, 2018

Administration and contact details

Host Department	Department of Mathematics
Host Institution	Macquarie University
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Subject details

Handbook entry URL	http://handbook.mq.edu.au/2018/Units/ResearchUnit/MATH702
Subject homepage URL	
Honours student hand-out URL	
Start date:	26 February 2018
End date:	1 June 2018
Contact hours per week:	2 hours
Lecture day and time:	TBA
Description of electronic access arrangements for students (for example, WebCT)	Material will be emailed to students.

Subject content

1. **Subject content description**

This unit provides an advanced introduction to key areas of research interest in modern algebra. It will centre around the theory and applications of modules over a ring. Modules are a common generalisation of the notions of vector space over a field, of abelian group, of

group representation, and of square matrix. We will see how to extend some of the theory of these notions developed in undergraduate years to the setting of modules. An important recurring idea will be that of a structure theorem, such as the undergraduate-level result that every finitely-generated abelian group is a direct sum of cyclic groups. We shall see various structure theorems for the various algebraic notions studied, with an important example being the Wedderburn theorem for semi-simple rings. Applications to representation theory will be particularly emphasised.

2. Week-by-week topic overview

The following topics will be covered.

- **Introduction:** Structure theorems for vector spaces, finite groups, finitely-generated abelian groups. First isomorphism theorem. Simplicity. Representations.
- **Rings and Modules:** Definitions and examples of modules over different rings. Homomorphisms, kernels, quotients, and direct sums of modules.
- **Zorn's lemma:** Posets, Zorn's lemma, the axiom of choice, bases for infinite-dimensional vector spaces.
- **Structure theory of modules over a semi-simple ring:** Simple modules, Schur's lemma, Maschke's theorem
- **Wedderburn's theorem:** a structure theorem for semi-simple rings
- **Applications to representation theory.**

3. Assumed prerequisite knowledge and capabilities

Linear algebra: vector spaces, linear transformations, bases, rank-nullity theorem, eigenvalues, and diagonalization.

Group theory: homomorphisms, subgroups, quotient groups; preferably some familiearity with the First Isomorphism Theorem, and the structure theorems(s) for finitely generated abelian groups.

4. Learning outcomes and objectives

AQF specific Program Learning Outcomes and Learning Outcome Descriptors (if available):

AQF Program Learning Outcomes addressed in this subject	Associated AQF Learning Outcome Descriptors for this subject

K1	coherent and advanced knowledge of the underlying principles and concepts in one or more disciplines
K2	knowledge of research principles and methods
S1	cognitive skills to exercise critical thinking and judgement in developing new understanding
S2	cognitive and technical skills to demonstrate a broad understanding of a body of knowledge and theoretical concepts with advanced understanding in some areas
S3	cognitive skills to exercise critical thinking and judgement in developing new understanding
A1	with initiative and judgement in professional practice and/or scholarship
A2	to adapt knowledge and skills in diverse contexts

Learning Outcome Descriptors at AQF Level 8
Knowledge
K1: coherent and advanced knowledge of the underlying principles and concepts in one or more disciplines
K2: knowledge of research principles and methods
Skills
S1: cognitive skills to review, analyse, consolidate and synthesise knowledge to identify and provide solutions to complex problem with intellectual independence
S2: cognitive and technical skills to demonstrate a broad understanding of a body of knowledge and theoretical concepts with advanced understanding in some areas
S3: cognitive skills to exercise critical thinking and judgement in developing new understanding
S4: technical skills to design and use in a research project
S5: communication skills to present clear and coherent exposition of knowledge and ideas to a variety of audiences
Application of Knowledge and Skills
A1: with initiative and judgement in professional practice and/or scholarship
A2: to adapt knowledge and skills in diverse contexts
A3: with responsibility and accountability for own learning and practice and in collaboration with others within broad parameters
A4: to plan and execute project work and/or a piece of research and scholarship with some independence

5. Learning resource

Lecture notes, supplementary materials, and suggestions for further reading will be provided.

6. Assessment

Exam/assignment/classwork breakdown					
Exam (take-home)	50 %	Assignment	50 %	Class work	Enter %
Assignment due dates		TBA			
Approximate exam date				12-15 June 2018	

Institution Honours program details

Weight of subject in total honours assessment at host department	12.5% of BPhil
Thesis/subject split at host department	BPhil has no thesis; thesis is 90% of MRes year 2
Honours grade ranges at host department:	
H1	85%
H2a	75 %
H2b	65 %
H3	50 %