Subject Information Guide

Convex and Nonsmooth Optimisation

Semester 2, 2016

Administration and contact details

Host Department	**Mathematical Sciences**
Host Institution	**RMIT University**
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Subject details

Handbook entry URL	Click here to enter text.
Subject homepage URL	Click here to enter text.
Honours student hand-out URL	Click here to enter text.
Start date:	**26/07/2016**
End date:	**25/10/2016**
Contact hours per week:	**2**
Lecture day and time:	**Tuesday 2.30 to 4.30pm**
Description of electronic access arrangements for students (for example, WebCT)	**Dropbox folder (TBA)**

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Subject content

1. Subject content description

Convex analysis refers to an area an area of mathematics that can be thought of as an extension of real analysis or some elements of functional analysis. Convex analysis is a special but important and plank of the larger field of nonsmooth analysis. Here we concern ourselves with functions that (in the main) evaluate to a real value but we do not assume that a derivative exists. Still we wish to be able to embark on the analysis of such function using a substitute for a derivative (call a subdifferential). In convex analysis we impose restriction on the geometry that these functions may possess. Indeed in convex analysis we associate function with sets and treat sets as the primary objects of this theory. This has the unexpected reward of allowing us to relax other assumptions usually associated with the theory of functions of several variables. These are:

1. Differentiability

2. Finiteness

The success of this approach has inspired similar theories for more general function classes and the development of variational theories which handle convergence issues in terms of sets, rather than function (called variational analysis). This greatly increases the usefulness of these theories and has given rise a wealth of mathematical techniques that have had wide application, especial in the areas of optimisation, control, economic modelling and other areas mathematics. In convex analysis and variational analysis we find a treasure chest of techniques and a mature calculus that facilitates its application and we can only be certain that more applications will arise in the future. In this course we will focus on the application of these ideas to problems arising in optimisation and related areas.

In the course we will:

1. Cover some of the fundamental theory that convex analysis is based on.

2. Show how these techniques can be applied to a number optimization problems.

3. Give an introduction to variational analysis, set convergence notions and the implications this as for convergence of functions, value functions and parametrised (or perturbed) optimisation problems.

4. Give and introduction to the some aspects of nonsmooth analysis and its application to optimisation problems and algorithms.

Despite this theory being well developed in infinite dimensions we restrict attention mainly to the finite dimensional case in this course. It is hoped that courses of this kind will alert more practitioners of applied mathematics to opportunities awaiting those who become proficient with the mathematical language of convex analysis.

2. Week-by-week topic overview

The following is a wish list that will be adjusted according to student's interests and the schedule as we go.

Week 1: What is Convexity? Operations that preserve convexity. Fundamental notions.

Week 2: Some basics on norms, inner products, spectral decompositions. Frobenius norms and the space of symmetric matrices. Adjoint operators.

Week 3: Support functions, separation theorems and subdifferentiability. Theorems of the alternative.

Week 4: Fenchel conjugate and its subdifferentiability. Fenchel conjugation calculus.

Week 5: Fenchel Duality and minimum norm problems. Subdifferential calculus. Examples.

Week 6: Duality of Linear systems. Linear programming duality. Positive semi-definite programming duality. Examples.

Week 7: Lagrangian duality, penaly methods, augmented Lagrangians and proximal point methods.

Week 8: Application to monotropic programming and networks. Integer programming.

Week 9: Introduction to subgradient Optimisation. Examples and applications such as compressed sensing.

Week 10: Set convergence. Epi-convergence and its consequences. Some convergence calculus. Implications for perturbations of optimisation problems.

Week 11: Introduction to nonsmooth analysis, Subdifferentials and calculus

Week 12: Infinal regularisations and special structures in nonsmooth analysis i.e. Prox-regularity.

3. Assumed prerequisite knowledge and capabilities

We will assume students have taken courses in real analysis, in linear algebra and a course on functions of several variables. For those students interested, the opportunity to engage in some programming in the Matlab environment for assessment purposes will be provided (this is optional). For these activities some experience in writing code in Matlab will be assumed.

4. Learning outcomes and objectives

Advanced knowledge of the techniques of convex and nonsmooth analysis. Ability to apply the calculus rules of the Fenchel conjugate and subdifferential calculus to formulate and reformulate optimisation problems. Be able to apply duality methods to derive equivalent dual problems. Be able to analysis the stability and convergence of perturbed optimisation problems. Appreciate the wide range of fields and problems to which the techniques from convex and nonsmooth analysis can be applied.

AQF specific Program Learning Outcomes and Learning Outcome Descriptors (if available):

AQF Program Learning Outcomes addressed in this subject	Associated AQF Learning Outcome Descriptors for this subject
Problem Solving - You will have the ability to apply knowledge and skill to characterise, analyse and solve a wide range of problems.	S1: cognitive skills to review, analyse, consolidate and synthesise knowledge to identify and provide solutions to complex problem with intellectual independence S2: cognitive and technical skills to demonstrate a broad understanding of a body of knowledge and theoretical concepts with advanced understanding in some areas A2: to adapt knowledge and skills in diverse contexts
Insert Program Learning Outcome here	Choose from list below

Learning Outcome Descriptors at AQF Level 8
Knowledge
K1: coherent and advanced knowledge of the underlying principles and concepts in one or more disciplines
K2: knowledge of research principles and methods
Skills
S1: cognitive skills to review, analyse, consolidate and synthesise knowledge to identify and provide solutions to complex problem with intellectual independence
S2: cognitive and technical skills to demonstrate a broad understanding of a body of knowledge and theoretical concepts with advanced understanding in some areas
S3: cognitive skills to exercise critical thinking and judgement in developing new understanding
S4: technical skills to design and use in a research project
S5: communication skills to present clear and coherent exposition of knowledge and ideas to a variety of audiences
Application of Knowledge and Skills
A1: with initiative and judgement in professional practice and/or scholarship
A2: to adapt knowledge and skills in diverse contexts
A3: with responsibility and accountability for own learning and practice and in collaboration with others within broad parameters
A4: to plan and execute project work and/or a piece of research and scholarship with some independence

5. Learning resources

Latexed lecture notes will be supplied during the course. If needed Matlab code will also be supplied to facilitate the learning of this computational environment.

6. Assessment

Exam/assignment/classwork breakdown					
Exam	60%	**Assignment**	40%	**Class work**	NA
Assignment due dates		TBA	TBA	TBA	TBA
Approximate exam date				To be negotiated with students	

Institution Honours program details

Weight of subject in total honours assessment at host department	**12.5%**
Thesis/subject split at host department	**25% thesis/75% course work**
Honours grade ranges at host department:	
H1	**80-100 %**
H2a	**75-79 %**
H2b	**70-74 %**
H3	**65-69 %**